



Paper to be presented at the
35th DRUID Celebration Conference 2013, Barcelona, Spain, June 17-19

The evolution of R&D networks

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Abstract

This paper models the formation of R&D networks in an oligopolistic industry. In particular, it focuses on the coevolution involving firms technological capabilities, market structure and the network of technological agreements. The main result of the paper is that the R&D network can work as a selection mechanism in the industry, creating ex post asymmetries among ex ante similar firms. This is due to a self-reinforcing process, in which events in the early stages of the industry affect firms survival in the long run. The nature of the technological environment affects the speed of the transition and the characteristics of the industry in the long run.

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1. Introduction

Recently, interfirm technological agreements have played an important role in the innovative activity of high-tech industries. More and more innovation is the result of joint R&D efforts and information sharing among firms: Powell *et al.* (1996) talked about “the network (of collaborating firms) as the locus of innovation”. The shortening of the product life cycle, the increased competition and the complexity of the knowledge base required for innovation force firms to cooperate even in one of the fundamental source of competitive advantage.

From the empirical point of view, an impressive number of studies in the fields of sociology, economics and business have thrown light on this phenomenon, although in a unsystematic way [see Powell and Grodal (2004) for a recent survey].

From a theoretical point of view, a rich literature in the game theoretic industrial organization tradition has discussed the effects of R&D cooperation [Katz (1986); d’Aspremont and Jacquemin (1988); Kamien *et al.* (1992)], while a small but growing number of works have studied the dynamics and the effects of technological networks within an evolutionary framework [Gilbert *et al.* (2001)].

The present contribution falls within the theoretical literature, proposing a model that focuses on the dynamics of R&D network formation. The model is inspired by the recent papers by Goyal and Moraga (2001) and Goyal and Joshi (2003), which apply the tools of network games introduced by Jackson and Wolisnki (1996) to study the

formation of R&D networks in a static framework. Our model extends their analyses, considering explicitly the dynamic feedbacks between market competition and firms' incentives to engage in collaboration.

The goal of the model is to derive propositions involving the joint dynamics of R&D network and market structure in the context of a model embodying as assumptions some of the evidence on interfirm technological agreements. Both empirically and theoretically, the study of this coevolution seems a promising direction to pursue, basically missing in the current state of the literature.

The main result of the paper is that the R&D network can work as a strong selection mechanism in the industry, creating *ex post* asymmetries in *ex ante* similar firms. This is due to a self-reinforcing, path-dependent process, in which events in the early stages of the industry affect firms' survival in the long run. In this framework, both market and technological externalities created by the formation of cooperative agreements play a role. Although it creates profound differences at the beginning, which are reflected by an unequal distribution of links across firms, the R&D network tends to eliminate them as it becomes denser and denser. The nature of the technological environment affects the speed of the transition and some of the characteristics of the industry in the long run.

The rest of the paper is organized as follows. Section 2 describes the model, whose analytical properties are derived in section 3. Section 4 presents results from numerical simulations. Finally, section 5 concludes.

2. The model

2.1 An informal description

Informally, the model can be summarized as follows. We consider the evolution of an industry where firms can introduce process innovations only through bilateral collaborations in an R&D activity, while remaining competitors in the market side¹. Firms produce a homogenous product, but they are different from the technological point of view: they have different levels of efficiency, which result in different levels of production costs, and different technological specializations, which allow complementarities to be exploited when firms collaborate.

We consider a discrete sequence of periods $t=0,1,2\dots$. Each period can be divided in two sub-periods: the *networking* phase, where firms can modify the network structure according to a procedure described below, and a *market competition* phase, where firms, given the network structure, compete in the product market. Competition is *à la* Cournot, so that firms' different production costs are reflected in firms' different performances. Firms' efficiency level is the result of the history of R&D collaborations for each firm. R&D collaborative projects are modeled as pairwise relationships: for each pair of firms involved in a collaborative agreement, the cost of the project is assumed to be fixed, while its effect (a deterministic reduction in the production cost) depends upon the technological profiles of the two firms.

In the networking phase of each period, two firms are randomly drawn to change the current state of their pairwise relationship, *leaving the state of the remaining R&D*

¹ We rule out the possibility of mergers (for instance, for antitrust reasons).

network unaltered. Two firms that are not collaborating can start a collaboration; two firms that are collaborating can decide to interrupt it. Capturing the bounded rationality of agents facing a complex evolution of network and technological capabilities, firms' decisions are based on the short run consequences on their profits. The resulting network for that period determines firms' level of efficiency, firms' technological specializations and firms' performance, which will constitute the new initial conditions for the subsequent period.

2.2 Firms and market competition

We consider a market where n firms produce a homogenous product. However, firms are *heterogeneous* from the technological point of view. They are located in a bi-dimensional technological space, and they are identified by the vector (γ_i, α_i) . $\gamma \in [\gamma_0, 1)$ is a parameter measuring the productive efficiency of a firm. It determines the unit cost of production according to:

$$c_i = c(1 - \gamma_i) \tag{1}$$

$\alpha_i \in (0, 1)$ characterizes the technological position of a firm, to be intended as its *technological specialization*. We assume that α does not affect *directly* the level of unit cost of production, but it is crucial in determining the value of collaborations.

We will term (γ_i, α_i) as *firm i 's technological capabilities*. Firms move over time in the technological space, and this is the effect of the network structure. Furthermore, we

define $\gamma_t \in [\gamma_0, 1)^n$ as the n -dimensional vector of variable γ at time t for all the firms; similarly, $\alpha_t \in (0, 1)^n$ is the vector of all technological positions at t .

Inverse demand is assumed to be linear:

$$p = A - Q \quad (2)$$

where Q is the total quantity produced by firms.

Firms are characterized by zero fixed costs of production. Given c_{it} , gross profits² are given by $\Pi_{it} = (p - c_{it})q_{it}$. Competition is *à la* Cournot, and it is assumed that firms play the (unique) Nash equilibrium in the one-stage game³. This means that the quantity produced by each firm at time t is:

$$q_{it}^* = \frac{a - n_t c_{it} + \sum_{j \neq i} c_{jt}}{n_t + 1} \quad (3)$$

where $n_t \leq n$ is the number of active firms (i.e. firms producing a strictly positive quantity) at t . We define N_t as the subset of such firms.

For sake of simplicity, firms that are inactive at time t are supposed to *exit* the market, never to reappear. This in particular implies that at the beginning of period $t+1$ all their existing links with other firms are severed, and since period $t+1$ onward they are no longer considered in the algorithm for network evolution. The discussion below on such

² Gross is referred to the cost of R&D. See below.

³ The assumed functional forms of demand and cost function, together with $A > c(1 - \gamma_0)$, assure the existence and uniqueness of equilibrium in the Cournot game.

an algorithm will make this point clearer. In equilibrium, gross profits are given by $\Pi^*_{it} = (q^*_{it})^2$.

2.3 The effects of the R&D network

In each period t , following the networking phase, the industry is characterized by an R&D network g_t . We define a binary variable $g_{ijt} \in \{0,1\}$: when $g_{ijt} = 1$, a collaborative link exists between firm i and j at time t . The network $g_t \in \{0,1\}^{\frac{n(n-1)}{2}}$ is then a collection of states for the pairwise relationships among firms. We indicate with $g + g_{ij}$ the network obtained by replacing $g_{ij} = 0$ in a generic network g with $g_{ij} = 1$, and similarly with $g - g_{ij}$ we denote the network obtained by replacing $g_{ij} = 1$ with $g_{ij} = 0$. Furthermore we define $N_t(i) \equiv \{j \in N_t \setminus \{i\} : g_{ijt} = 1\}$, that is the set of firms that have a collaboration with firm i at time t .

Innovation is modeled as a deterministic reduction in the unit cost of production. A network structure corresponds to a list of collaborators for each firm. Suppose to take a generic firm i : for i , collaboration with firm j at time t has a specific value v_{ijt} . The economic interpretation is as follows: whenever $g_{ijt} = 1$, firms i and j start a new R&D project together at time t , which allows them to reduce their unit cost of production to an extent that is function of v_{ijt} .

Therefore, such a value captures the opportunities for firm i to “learn” as a consequence of collaboration with firm j . In this framework, we refer to the process of learning as a process of knowledge “recombination”, an idea that dates back to Schumpeter and has

been recently rediscovered also in formal models [e.g. Weitzman (1998)]. According to this interpretation, the creation of new knowledge relies on pre-existing knowledge (of the pair) as major inputs. In the model, firm i 's knowledge (i.e. its technological capabilities) is completely described by the vector (γ_i, α_i) . Being exposed to firm j 's knowledge in the collaboration, firm i recombines its knowledge and improves upon it to an extent that is increasing in firm j level of efficiency (which is taken as a proxy for learning opportunities) decreasing in firm i 's level of efficiency (capturing decreasing returns in learning) and depending on firms' relative technological positions according to a well specified function. After collaboration, firm's technological positions are modified after collaboration⁴.

This representation of the learning process has the advantage of parsimony, since the distribution of technological capabilities in the industry identifies both the outcome of market competition and the effects of technological collaboration.

More specifically, the value from collaboration is given by $v_{ijt} = f(d_t(i, j))\gamma_{jt-1}$. It is increasing in γ_{jt} , since the higher is the level of efficiency of the collaborator (the more it is "knowledgeable"), the more the firm can learn from it. It is also increasing in the value assumed by a function f , whose argument is given by the technological distance between firms, as defined by $d_t(i, j) = |\alpha_{it-1} - \alpha_{jt-1}|$. Nooteboom (1999) has argued that firms need to be technologically "not too distant, nor too near" for effective collaboration to take place. This is because there are two opposing forces: if firms are distant, their different technological specializations can create opportunities for complementarities and synergies; but if they are too distant, they lack the "absorptive

⁴ A similar representation of knowledge, in the context of knowledge creation as knowledge recombination, can be found in Cowan *et al.* (2004).

capacity” [Cohen and Levinthal (1989)] to learn from their collaborator and cognitive distance can harm effective communication. This conjecture has found empirical support [e.g., Mowery *et al.* (1998)] and it is reflected in the particular functional form chosen for f , which is assumed to be a concave parabola:

$$f(d_t(i, j)) = a_1 - \frac{a_2^2}{4a_3} + a_2 d_t(i, j) - a_3 d_t(i, j)^2 \quad (4)$$

$$a_1, a_2, a_3 > 0$$

$$f(d_t(i, j)) \geq 0 \forall d_t(i, j) \in [0, 1]$$

The vector (a_1, a_2, a_3) identifies the technological characteristics of the industry. $\frac{a_2}{2a_3}$

is the optimal technological distance, as the result of the counterbalancing forces of absorptive capacity and search for complementarities⁵. a_1 is a measure of “technological opportunities”, being $a_1 = \max_d f(d)$.

Given the total value of collaboration $V_{it}(g_t) = \sum_{j \in N_t(i)} v_{ijt}$, γ_{it} is determined by

$$\gamma_{it} = 1 - e^{-\lambda L_{it}} \quad (5)$$

where

$$L_{it} = L_{it-1} + V_{it}(g_t) \text{ and } \lambda > 0$$

Equation (5) captures the decreasing marginal returns in the innovative process.

⁵ Parameters are assumed to be chosen such that the maximum point lays in the appropriate interval.

Finally, we assume that through collaboration firms modify their technological position.

Formally:

$$\alpha_{it} = \rho\alpha_{it-1} + (1-\rho) \sum_{j \in N_t(i)} \frac{\gamma_{jt-1}}{\Gamma_{it-1}} \alpha_{jt-1} \quad \text{if } N_t(i) \neq \{\emptyset\} \quad (6)$$

$$\alpha_{it} = \alpha_{it-1} \quad \text{otherwise}$$

where $\Gamma_{it-1} = \sum_{j \in N_t(i)} \gamma_{jt-1}$ and $\rho \in (0;1]$.

The final technological position of a firm at time t is a linear combination of its old technological position and a weighted average of technological positions of collaborating firms. A collaborator is weighted more if it has a high efficiency level (which implies more opportunities of learning). When $\rho < 1$, firms become technologically more “similar” to their collaborators. When $\rho = 1$ (so that technological positions are time-invariant), firms maintain their technological specialization in the process of learning (when they recombine their knowledge).

2.4 The evolution of the network

Each period two firms among the ones still in the market are randomly chosen to possibly change their network state. Firms that are not currently collaborating can decide to form a collaborative link, firms that are already collaborating can sever the existing link. Each link has the same probability to be revised.

We assume that maintaining a collaborative link costs each firm a fixed amount $E > 0$ in each period. E has to be interpreted as the firm's contribution to the joint R&D project.

For a firm involved at time t in $|N_t(i)|$ collaborations, net profits are equal to $\Pi_{it} - |N_t(i)|E$.

The proposed algorithm can be reformulated as follows: each period, two firms are allowed to modify their portfolio of collaborations, starting a new collaboration between each other if it does not exist, or interrupting it if exists. The state of the remaining network is unaltered: all the other collaborations in which these firms are involved, and the collaborations of all the remaining firms are automatically confirmed. In other words, network at time $t-1$ and time t may differ only for the state of one link.

Suppose that at period t , the link ij (i.e. the potential or existing link involving firms i and j) is randomly chosen to be updated. Define $\Pi_{it}(g; \alpha, \gamma)$ as the profit for i resulting from market competition when the network is g and the initial technological capabilities are given by (α, γ) .

If $g_{ijt-1} = 1$, the link is severed if $\Pi_{it}(g_{t-1} - g_{ij}; \alpha_{t-1}, \gamma_{t-1}) > \Pi_{it}(g_{t-1}; \alpha_{t-1}, \gamma_{t-1}) - E$ or $\Pi_{jt}(g_{t-1} - g_{ij}; \alpha_{t-1}, \gamma_{t-1}) > \Pi_{jt}(g_{t-1}; \alpha_{t-1}, \gamma_{t-1}) - E$, while in the opposite case it is maintained. This means that a firm wants to sever an existing link if profits without the link are higher than the profits with the link minus the R&D cost. If $g_{ijt} = 0$, the link is

formed if $\Pi_{it}(g_{t-1} + g_{ij}; \alpha_{t-1}, \gamma_{t-1}) - E \geq \Pi_{it}(g_{t-1}; \alpha_{t-1}, \gamma_{t-1})$ and

$\Pi_{jt}(g_{t-1} + g_{ij}; \alpha_{t-1}, \gamma_{t-1}) - E \geq \Pi_{jt}(g_{t-1}; \alpha_{t-1}, \gamma_{t-1})$. If a link does not exist, it is formed

when for both firms the gain stemming from forming the link is higher than the R&D cost they have to sustain.

In order to avoid that with probability 1 no link is profitable at $t=0$, we assume that

$$E \leq E^*, \quad \text{where} \quad E^* = \left(\frac{A - (n-1)c(1 - \gamma_1^{\max}) + (n-2)c(1 - \gamma_0)}{n+1} \right)^2 - \left(\frac{A - c(1 - \gamma_0)}{n+1} \right)^2 \text{ and}$$

$$\gamma_1^{\max} = 1 - e^{-\lambda(L_0 + a_1)}.$$

In terms of behavioral assumptions, the proposed rule implies that agents are myopic, since they decide only on the basis of their current pay-off, but at the same time they have rational expectations *within a given period*, since during the networking phase at time t are able to predict correctly the marginal cost of their rivals at time t and the Nash equilibrium that will be played in the market phase.

This assumption of myopic behavior aims at the representing the bounded rationality of agents who face a highly complex and uncertain future evolution of the R&D network and of the technological capabilities of firms in the industry.

3. Analytical results

In this section we provide some analytical results. First, we consider the incentives to form collaborative links at the level of the *single pair* of firms. We will also show two numerical examples, for the set of parameters we will consider in the simulations. Then we will turn to the long run properties of the system. Although the stochastic process generated in the model is rather complex, a clear and intuitive result holds for the network state in the long run.

3.1 Firms' cooperative strategies

Let us introduce the following function:

$$F_i(\alpha_j, \gamma_j | (\alpha, \gamma)_{-j}) = \left(\frac{A - nc(1 - \gamma_i)(e^{-\lambda(\gamma_j f(d(i,j)))}) + c(1 - \gamma_j)e^{-\lambda f(d(i,j))\gamma_i} + \sum_{k \in i, j} c_k}{n+1} \right)^2 - \left(\frac{A - nc(1 - \gamma_i) + c(1 - \gamma_j) + \sum_{k \in i, j} c_k}{n+1} \right)^2 \quad (7)$$

Suppose to take a generic pair of firms i and j . Fix the technological capabilities of the other $(n-2)$ firms, and from $\gamma_k, k \in N \setminus \{i, j\}$ derive the unit cost of such firms. Studying $F(\cdot)$ we can answer to the following question: how does the gross gain (i.e. the variation in profits excluding R&D costs) for i of forming a link with firm j vary, as a function of j 's and i 's technological capabilities?⁶

In order to make computation easier, we write F as:

$$F_i(\alpha_j, \gamma_j | (\alpha, \gamma)_{-j}) = \left(\frac{c(1 - \gamma_j)(e^{-\lambda f(d(i,j))\gamma_i} - 1) - nc(1 - \gamma_i)(e^{-\lambda \gamma_j f(d(i,j))} - 1)}{n+1} \right) (q_i(+ij) + q_i(-ij)) \quad (8)$$

where $q_i(+ij)$ and $q_i(-ij)$ represent the quantities produced by firm i with and without the link with firm j respectively. The first factor represents a necessary condition for

⁶ Notice that we restrict our attention to the cases where the formation of the link does not lead to the exit of any firms.

collaboration: the net effect of counterbalancing forces on firm i 's profits given by the reduction in its costs and in firm j costs must be positive, i.e. firms must increase the quantity they produce (and consequently their profits). Consistent with the existence of an interior solution, firms i and j are assumed to be close enough in efficiency levels so that the necessary condition is always satisfied.

We can show that the following propositions hold (the proofs can be found in the appendix):

Proposition 1 *Ceteris paribus, gains from the collaboration increase when firms' technological distance move towards the "optimal technological distance", and decrease otherwise.*

Proposition 2 *Ceteris paribus, the effect of an increase of γ_j on the gains from the collaboration is ambiguous. Possibly, an inverse U relation holds between γ_j and gains from collaboration.*

Proposition 3 *Ceteris paribus, the effect of an increase of γ_i on the gains from the collaboration is ambiguous. Possibly, an inverse U relation holds between γ_i and gains from collaboration.*

Proposition 4 *Ceteris paribus, gains from the collaboration decrease when the remaining firms' average efficiency increases.*

The first proposition is obvious. Proposition 2 is more interesting. The rationale for the possibly non-monotonic relationship is straightforward, however. High efficiency of a collaborator is good for a firm since the opportunities of learning increase and the extent

the collaborator can learn from the firm is limited, but at the same time it is bad since efficiency is correlated with size. If a firm i 's potential collaborator is highly efficient, then it is “large”. This makes i a “small” firm, in relative terms. Since we consider process innovation, smaller firms have lower total gains per unit of cost reduction, and their incentive to collaborate and innovate, *ceteris paribus*, is smaller. This is the so-called “cost spreading” argument, which has been claimed to be one of the advantages in innovation by large firms and it has found empirical support [Cohen and Klepper (1996)].

The nature of the opposing forces is symmetric in Proposition 3. If firm i is highly efficient, it assures great opportunities of learning to its potential collaborator, and the reduction in its unit cost is smaller in absolute value. At the same time firm i is “large”: so that reduction in unit cost of production can be spread over a larger quantity.

Finally, the average efficiency of other firms (Proposition 4) comes into play through the usual channel: its effect on firm's size. Its increase decreases the gains from collaboration, since it makes the firm “smaller” in relative terms.

The results show the complex nature of the interaction between the technological and markets aspects concerning firms' incentives to collaborate. Furthermore they stress the feedbacks between firms' incentives and the evolution of the network. Network evolution affects firms' incentive to collaborate through market competition and opportunities for learning. In turn, the network changes according to firms' decision. Firms' strategies and the network *coevolve*, a point that has already been raised by business scholars [e.g., Koza and Lewin (1998)].

Figure 1 and Figure 2 show the behavior of $F(\cdot)$ under the parameterization of the “Standard Simulation” discussed in the next session. In the first case (Figure 1),

$\gamma_i = 0.35$ and $\sum_{k \neq i, j} \frac{\gamma_k}{n-2} = 0.35$. Firm i is sufficiently small so that the inverse U relationship between gains from collaboration and γ_j emerges. When γ_j is large enough (approximately 0.5), the negative effect on size prevails on the positive effect of technological opportunities. If instead $\gamma_i = 0.5$ (Figure 2), firm i 's size guarantees that an increase of γ_j monotonically increases the gains from collaborations.

INSERT FIGURE 1 ABOUT HERE

INSERT FIGURE 2 ABOUT HERE

3.2 *The long run properties of the system*

Although the stochastic process describing the evolution of the R&D occurs on a rather complicated state space, it is possible to derive clear results about the limit behavior of the network structure.

The industry at time t is completely characterized by the state $\{g_t, \gamma_t, \alpha_t\}$. Then, it is easy to verify that the underlying stochastic process satisfies the Markov property. Theorem 1, whose proof can be found in the appendix, concerns the long run properties of such a process.

Theorem 1 *As $t \rightarrow \infty$, each link is absent with probability 1. The absorbing states of the process are characterized by the empty network, and the set of these states is reached almost surely in the long run.*

The intuition behind this result is simple and comes directly from the existence of marginal decreasing returns in the outcome of collaboration. Since innovative opportunities become smaller and smaller as firms continuously invest in R&D, while its cost is constant and strictly positive, it will come a time where forming or maintaining collaborative links is not convenient, irrespectively of other firms' technological positions. Loosely speaking, when (“almost”⁷) everything that could be discovered has been discovered, investing in R&D becomes unprofitable. Nevertheless, we are mainly interested in the transition phase of the system, per se and for the way it affects the final equilibrium is reached. This will be the subject of next section, where numerical simulations of the model are reported.

4. Simulation results

In this section we discuss the results emerging from a series of numerical experiments performed with the model. Although several exercises are possible, the ones reported here are illustrative of the basic mechanisms underlying the model.

In the “Standard Simulation”, we consider a situation where competition is tough at the beginning. Market size is $A=65$, 16 firms populate the industry at time 0, and their initial unit cost is about 47.56 ($c=50$, $L_{i0} = 5 \forall i \in N$). The initial network is empty. The “optimal” technological distance is 0.25, and technological parameters are chosen in a way that the expected value of $f(d)$ is 0.5, ($a_1 = 0.56, a_2 = 0.5, a_3 = 1$), under the assumption of technological positions that are uniformly distributed along the interval (0,1). The R&D cost is “high”, $E=0.0230$, and corresponds to $0.975 * E^*$, where

⁷ As the simulation will make clear, the precise quantification of “almost” is endogenous to the model.

E^* is the largest R&D cost for which firms at optimal distance will form a link given their initial costs. $\rho = 1$, so that technological positions are time-invariant. We run the experiments for 1000 periods, by which a steady state is reached.

Figure 3 and 4 reports the results for the average of 40 replications. As for all the figures in the paper, bars indicate the 95% confidence interval for the true population statistics.

INSERT FIGURE 3 ABOUT HERE

INSERT FIGURE 4 ABOUT HERE

Figure 3 reports the number of active firms over time⁸. The number of active firms has a sudden drop around period 45: a shakeout occurs. In the steady state, less than 8 firms on average are in the market, then slightly less than half of the initial number of firms. The shakeout (defined as a significant and rapid reduction in the number of firms active in the market) is a typical feature of the evolution of industries in early stages, as represented by the theory of industry life cycles [Klepper (1997)]. In the model, it is the process of network formation that creates the shakeout among firms that are symmetric at the beginning. In other words, the existence of a R&D network (i.e. the possibility for firms to form cost-reducing links) operates as a strong *selection mechanism*.

Figure 4 further elaborates on this point, and shows an interesting dynamics involving market structure and the network of collaborating firms.

⁸ Figure 3 reports the average number of firms active in each period across simulations. For this reason, we observe fraction of firms.

The figure reports the dynamics of three variables: total output produced in the market, normalized by market size ($\frac{Q}{A}$); market concentration, measured by the Herfindhal index; and network density, which is the fraction of existing links over the total number of possible links (considering only the firms still in the market). The scales of these variables are different. For preserving readability and comparison of behavior over time, total quantity and density are to be read along the left axis, while the right axis is for the Herfindhal index.

The evolution of the industry can be described as follows. At the beginning the density of the network is growing relatively slowly. Since R&D costs are relatively high, market relatively small and the average level of efficiency in the industry low, firms need to find partners located almost at the optimal technological distance, and this process is assumed not be instantaneous. This creates differences in the relative competitiveness of firms, expressed by a sharp increase in the concentration index. However, given the low average level of efficiency in the market, the process of “knowledge recombination” is reflected by a limited growth rate for total output, which, given the assumptions, is only depending on the *average* efficiency of firms.

When the shakeout occurs, the time series for the network density has a break: this is due to the fact that the firms exiting the market have typically no links, and then they lowered the average number of links. However, the process of links formation continues, until a *complete* network (density 1) emerges for around 100 periods. Concentration continues to grow, but then it starts declining when the density reaches a sufficiently high level: the network operates first as a mechanism creating different

efficiency levels and then as a mechanism favoring the “catching-up” of relatively less efficient firms⁹.

For around 100 periods, therefore, we can observe a sort of “equilibrium”, where almost equal size firms operate in a complete network.

The behavior of total output, reflecting the behavior of average efficiency, follows an S-shaped curve. The growth rate of total output is the highest during the formation of the network after the shakeout. The increasing density of the network, the increase in the average level of efficiency (which creates more opportunities for recombination) and the fact that marginal decreasing returns are not limiting innovative opportunities yet, generate a high rate of growth in the average efficiency level. Interestingly, the inflection point in the output series roughly corresponds to the time in which a complete network is formed. Then, the “equilibrium” in market structure and network dynamics is accompanied by a low growth of the average efficiency level.

Since we model innovation as a process in which knowledge is both an input *and* an output, we interpret the results in the following way: in the early phases network formation mainly drives the creation of knowledge, in the late stage it is the existence of a large pool of knowledge which preserves the incentive for firms to form new links (i.e. the cause-effect relation between network formation and knowledge creation is reversed while time elapses).

The final period occurs when technological opportunities have substantially been depleted. Total output and market shares stabilize, and firms start to remove all their links. The final long run equilibrium is then reached when the empty network is finally obtained.

⁹ This result is clearly associated to the asymptotic nature of the cost function: knowledge is always created, if a firm is connected, but at a decreasing rate.

It is also interesting to look at the evolution of the network structure over time, especially for the phase immediately preceding and following the shakeout.

Figure 5 reports the behavior of the group degree centralization index over the simulation time. This index takes a firm's degree (its number of links) as its centrality measure, and it basically summarizes how the links are distributed across firms. It takes value 0 when all the firms have the same number of links (as it happens in a regular network, like the complete network), and value 1 in a star, where there is one firm connected to all the others, and no other links exist (Wasserman and Faust, 1994).

Formally, define as n_{it} the number of links that firm i has at time t , and $n_t^* = \max_i n_{it}$.

The group centralization index is defined as follows:

$$C_D(t) = \frac{\sum_{i \in N_t} (n_t^* - n_{it})}{[(n_t - 1)(n_t - 2)]} \quad (9)$$

INSERT FIGURE 5 ABOUT HERE

The index shows a marked growth until the shakeout period: this implies that in this phase links are more and more unequally distributed across firms. Then, the value of the index falls down in a similar way, to reach the value of zero when the network becomes complete. Then it naturally grows again, when firms start removing their links, and comes back to zero, when the network is empty.

4.1 Discussion

Two results deserve further explanations. The first point is that, even if firms are symmetric *ex ante*, the opportunity of forming R&D links can generate profound asymmetries *ex post*: in the long run, these are reflected in firms' survival.

Table 1 reports the statistics concerning the number of links for firms at period 40 (just before the shakeout) and their survival in the long run. The variable *link40* takes value 1 if the firm has at least one link at period 40; the variable *surviving* has value 1 if the firm survives the shakeout.

INSERT TABLE 1 ABOUT HERE

The table clearly shows that firms exiting the market are firms without links. Furthermore, an inspection of the network structure in the initial phase shows that the network structure, at the shakeout, is typically given by a single component of connected firms, while remaining firms are disconnected. A first strong selection occurs between firms that *are* in network, and survive at the first shakeout, and firms that are not "able" to join the network "reasonably" soon. The fact that firms without links eventually exit the market is obviously not surprising, since it is the natural consequence of the assumption that costs are reduced only through collaborations. The interesting point is the mechanism through which some firms are *excluded* by the R&D network.

Second, we need to explain also the evolution of the network structure, in particular the increase in centralization in the initial phase. The firms' polarization in two groups of

connected and disconnected firms is a candidate for a first basic explanation, but the evolution *within* the main component can also be an important determinant.

We will show that both the selection process and the evolution of the network structure are driven by a self-reinforcing, path-dependent process, in which events in the early stages of industry affect firms' centrality in the initial network with long term consequences in terms of survival (Arthur, 1990). Forming links at the beginning (which in the model is due to random factors, and in the real world could correspond to different managerial practices, social contacts or other small "historical accident" affecting firms' networking propensity) propels a positive feedback mechanism that favors the centrality of such firms, and entraps excluded firms in their status. However, *among the surviving firm*, the negative feedbacks end up prevailing, and firms converge in market shares and efficiency levels.

The first mover advantage of firms forming links at the beginning comes from the net effects of forces described in the previous section. Firms that are "lucky" and form links in the first periods become larger than the other firms. This increases their incentive to form new links, since in this phase decreasing returns are not substantial yet. At the same time, large firms are more efficient and competent (they are larger *because* they are more efficient) and they offer their collaborators more opportunities to learn. A complementarity exists between "large" and "small" firms: large firms are willing to cooperate because of the "cost spreading" argument and because of the search for technological complementarities; small firms are willing to collaborate because of the high level of competences they can find in large firms. The final effect of this process is the tendency to reinforce the centrality of first movers firms, which results in the sharp increase of the centralization index. This process comes naturally to an end since the

number of possible links to be formed is limited. This corresponds to the phase of industry maturity, when the network becomes complete.

At the same time, firms that are not able to form links in the initial phase are excluded by the subsequent process of the network formation: their incentive to start collaborations decreases because such firms are getting smaller and smaller, and they are a limited source of learning opportunities for their potential collaborators.

Overall, this suggests an industrial structure where one can identify three kinds of firms, identified by their position in the network in the initial phase: 1) central actors, whose position is strongly path-dependent and that can gain a (temporary) leadership in the market; 2) (temporarily) peripheral actors, that is firms that are active in the network in relatively laggard positions, but are destined to catch up with the leader, if able to survive the shakeout; 3) isolated firms, which are not able to join the network soon, being trapped in a self-reinforcing mechanism of exclusion, and which end up exiting the market.

For a quantitative assessment, we run two OLS regression on the data generated by the simulations. We considered the variation on the number of links between period 40 and period 10 as dependent variable (*newlink40*), and we regressed it on the number of times a firm has been called to change its network status from period 10 to period 40 (*newcalled40*) and on the number of links the firm have at period 10 (*link10*). In a sparse network, the first variable is clearly supposed to have a positive coefficient. Table 2 shows that, at the beginning of the life cycle, also the sign of the coefficient for the second variable is positive, and significant. We have the confirmation that the “Matthew effect”¹⁰ is at work here: firms that are more central at the beginning are more

¹⁰ The term refers to the Gospel According to St Matthew: “For unto every one that hath shall be give, and shall have abundance: but from him that hath not shall be taken away even that which he hath”.

likely to attract new collaborators in the following periods. This property is often found in networks of alliances [see, for instance, Powell *et al* (1996)].

INSERT TABLE 2 ABOUT HERE

Concerning the selection process, the picture so far must be enriched including the role played by the externalities arising in the process of network formation. When two firms form a link, they always create a negative externality upon the remaining firms (“business stealing” effect). However, when two firms start to collaborate, this also creates a positive “technological externality”, but only for firms connected with these two firms. The new projects increase the rate of growth of efficiency of the two partners, with a positive effect on the technological opportunities for their collaborator. The increasing network density is strongly penalizing for firms outside the active network at the beginning, since they find increasingly difficult to join the network.

It is interesting to compare our results with the static analysis by Goyal and Joshi. In a similar setting with Cournot competition, *ex ante* identical firms and fixed costs for link formation, they show that only three network structures can be sustained as equilibria, as a function of the level of the link formation cost: the empty network, the complete network and the dominant group network, in which only a non singleton, fully connected component is formed. Our model generates as a long run structure exactly a dominant group network, and it provides a dynamic mechanism that leads to this structure. This emphasis on asymmetric networks, which have profound effects on market structure, is consistent with the empirical evidence on the firms’ motivation to engage in collaboration [Hagedoorn, (1993)].

In commenting the results, there is an important final remark that has to be done. The results of the model are not purely dependent on the randomness associated to link revision. In particular, the shakeout is not simply driven by the fact that some firms are not drawn to form links. Randomness plays a role because it perturbs an initially symmetric situation, giving some firms an initial advantage. After that, an economic self-reinforcing mechanism operates, which significantly reduces the role of randomness. In other words, the model shows the instability of a symmetric market structure, when firms can form pairwise links. For this reason, it seems reasonable to start with an empty network in a symmetric set-up (in terms' of efficiency levels). If the network at time $t=0$ were a non empty network (for instance, a random graph), this would guarantee some firms (the firms with more collaborations at $t=0$) an exogenously given advantage, which would increase the probability of such firms to become central actors in the evolution of the network. The same argument applies if one removes the assumption of equally efficient firms at $t=0$. Furthermore, as section 4.3 will show, the results do not depend on isolated firms being fixed in their level of efficiency.

4.2 Comparative dynamics

A natural question concerns possible exercises of comparative dynamics. In theory, several different parameterizations can be discussed. Here, we consider two of them.

First, we increase technological opportunities. a_1, a_2, a_3 are chosen in a way that the expected value of $f(d)$ becomes 0.75 (instead of 0.5)¹¹. We call this experiment Simulation A. The opportunities for “knowledge” recombination within collaborative projects increase, making collaboration more attractive, *ceteris paribus*. Notice that high opportunity here does not mean that there is “more” to learn in the long run (unit cost is bounded from below, and it always (potentially) converges to 0), but that learning is easier. The effect on market structure is ambiguous, *a priori*. On one hand, more firms can engage in collaboration, especially at the beginning. On the other hand, the average efficiency growth rate is expected to be higher, and this is detrimental for the survival of firms that do not join immediately the network. As figures show, both effects are at work: with “high opportunities”, the equilibrium number of firms is higher (the long run level of concentration is lower), but the shakeout occurs typically earlier. Technological progress is faster, as expected. Notice, finally, that the network does not reach density 1. This is easily explained by the fact that the faster depletion of innovative opportunities makes inconvenient the formation of links *before* a complete network is reached.

INSERT FIGURE 6 ABOUT HERE

Until now, we considered time invariant technological positions. Empirical evidence [Mowery *et al.*] suggests that interfirm technological agreements are important in explaining the movement of firms over time, and they can lead firms to become technologically more similar at the dyadic level.

¹¹ In particular, $a_1 = 0.84375$, $a_2 = 0.75$, $a_3 = 1.5$

In order to study the impact of variation of ρ on network evolution, we consider the case where technological heterogeneity matters in the outcome of collaboration, fixing the optimal distance at 0.5 (but keeping fixed the expected value of f)¹².

For this case, we run two sets of simulation, one with $\rho = 1$ (Simulation B), the other with $\rho = 0.99$ (Simulation C).

The results are reported in Figure 7 and 8.

INSERT FIGURE 7 ABOUT HERE

The first remark concerns the comparison between Simulation B and the Standard Simulation. Although the qualitative picture is rather similar, one can observe a slightly higher level of concentration in the long run. This is due to the relationship between the optimal distance and the initial distribution of technological positions. It is intuitive to see that, once the assumption of uniformly distributed firms is maintained, increasing the optimal technological distance over a certain threshold makes less likely for firms to find a partner at the optimal technological distance, especially for firms with intermediate technological positions. Since at the beginning this is what really matters, more frictions are introduced in the search of a satisfying partner. Firms lucky enough to find such partners get a stronger advantage. Progress is less rapid, concentration is higher and the network less dense. This is clearly an example which shows that the hypothesis on the initial distribution of firms matters, especially for certain technological environments, because it affects the opportunity for cooperation in the industry. This aspect deserves further analysis in the future.

¹² $a_1 = 0.5517$, $a_2 = a_3 = 2.2069$

INSERT FIGURE 8 ABOUT HERE

The changes when ρ is smaller than 1 are radical. At the beginning the evolution is the same. This is not surprising, since the process of technological convergence takes time. The difference occurs after the shakeout. The process of network formation soon comes to an end. The reason for that is simple: the emergence of one single component inevitably lead to the overall convergence to a single technological position, which is detrimental for innovation. In the forty replications, the final value of the average technological distance lies in the interval $[0.002,0.02]$. This implies that both technological progress and convergence in market shares stop.

This result shows the important role that entry, a factor not considered in the model, can play. In a relatively mature industry, in which the technological positions of incumbents have converged, new entrants have an important role to play. They can bring into the market different capabilities. This also help the new firms to survive, although less efficient, because of their role in the network. Extending the model to the role of new entrants is an interesting exercise that we plan to realize.

4.3 Extensions

In this section we check the robustness of the results with respect to two main assumptions of the models. First, we implement two other algorithms driving the formation of the R&D network; second, we introduce, although in a simple way, an

alternative source for cost reduction. Overall, the model exhibits robustness with respect to these changes.

Concerning the rules for links revision, it has been maintained the hypothesis of revision of *one* link per period. Given this restriction, two different algorithms have been considered. The first one can be defined as “socially oriented”, and it aims at capturing the idea that meetings are more likely between firms that have collaborators in common.

In practice, the algorithm works as follows:

- a) One firm is picked up randomly. Each firm has the same the probability to be chosen.
- b) With probability $\frac{|N_i(t)|}{n-1}$ the firm revises the state of one of its existing links; otherwise, the firms revise the state of one of its non-existing links.
- c) In the case of revision of an existing link, a firm $j \in N_i(t)$ is chosen with uniform probability.
- d) In the case of revision of a non-existing link, a given firm j is chosen by i to revise the state of the link with probability:

$$\frac{1 + |N_i(t) \cap N_j(t)|}{\sum_{k \in N_i(t)} 1 + |N_i(t) \cap N_k(t)|} \quad (10)$$

i.e. the probability of "meeting" is increasing to the number of collaborators that the two firms have in common.

The second algorithm will be labeled as “economically oriented”. It is meant to capture the active, “rational” firm’s search for optimal partners.

The algorithm works as follows:

- a) One firm is picked up. Each firm has the same the probability to be chosen.
- b) For each $k \neq i$, net profits for i resulting from the meeting with k are computed. In particular, if the link ik does not exist, firm i correctly predicts the willingness of k to cooperate or not. We indicate with $\Pi_i(ik)$ such profits.
- c) The firm j that is chosen is given by $j \equiv \arg \max_{k \in N_i(t)} \Pi_i(ik)$. In case of ties, the firm with the highest index is chosen.

Figure 9 and 10 reports the Herfindhal index, the total output and network density for the same parameterization of the “Standard Simulation”, when the algorithms of network formation are respectively the “socially” oriented algorithm (Simulation D) and the “economically” oriented-one (Simulation E).

INSERT FIGURE 9 ABOUT HERE

INSERT FIGURE 10 ABOUT HERE

The effects of the “socially” oriented algorithm are negligible. The results are easy to interpret. What is crucial in the model are the first links formed, when the self-reinforcing mechanism is at the work. Since at the beginning the network is sparse, the probability of meeting is basically uniform, and the differences are necessarily of minor

importance. When the network has reached a sufficiently high density (i.e. in the periods just preceding the shake out), firms active in the network become significantly more likely to meet. But these firms are also the more likely to be willing to start cooperation, since they are larger and more competent. The effect, then, is simply to make the convergence towards the complete network slightly more rapid, and consequently the shakeout slightly more rapid, without an impact on the qualitative behavior of the system.

The “economically oriented” algorithm has instead a more significant effect. This is similar to an increase in technological opportunities: the shakeout occurs earlier, but involves fewer firms. This algorithm substantially reduces the frictions in the network formation, and then it leads to a stronger role of the first mover advantage. Larger firms at the beginning have more incentives to form new agreements, and they are preferred as partners, *ceteris paribus*, because they provide more learning opportunities for their collaborator; small firms can look for the largest firms. In this way, more links are formed: the shakeout is anticipated (because the exclusion process starts in advance) but involves fewer firms. In any case, the selection process is strong.

In terms of alternative sources of cost reduction, a simple formulation has been considered. We relax the assumption that costs can be reduced only through collaboration. Each period, each firm is assumed to start an "in-house" R&D project. More generally, other factors (for instance, learning by doing) can lead to such a reduction in costs. The assumption is that this process of cost reduction does not require any investment by the firm. Introducing explicitly an R&D cost (e.g. a fixed cost similar to the costs required for cooperative R&D) in the framework of a simultaneous game would create a problem of multiple equilibria, when firms are close enough in efficiency

level. Even if one assumed some rule to pick up one equilibrium, this would be too complex to implement.

In economic terms, this assumption can be justified by claiming that collaborative projects are typically started for larger, more costly (and with higher benefits) projects than in-house R&D. The assumption of no cost approximates a situation where each firm can always cover the costs of internal R&D, and the costs can be consequently not modelled. Furthermore, in the present context, we introduce in-house R&D to check the robustness of the results, and not to fully model the choice between in-house and cooperative R&D. Here, one major point is to check the robustness of the selection result due to the network formation. With positive costs (and indivisibility), small firms would not invest in R&D alone either (for the cost spreading argument). Then the “no cost” situation can be interpreted an upper bound for outcome of the selection process: selection cannot be stronger than the case of “costless” R&D.

Following the notation of the paper, we label v_{iit} the value of such an in-house project.

We consider two possible formulations:

$$v_{iit} = \beta f(0) \gamma_{it} \tag{10}$$

$$v_{iit} = \beta f(0) k \tag{11}$$

In the first case (Simulation F), we consider a cumulative process: more competent firms have more valuable in-house projects; in the second case (Simulation G), instead, the value is independent from firm's level of efficiency. This second case is clearly more favorable to 'laggard' firms, and it is introduced mostly as a benchmark case.

INSERT FIGURE 11 ABOUT HERE

INSERT FIGURE 12 ABOUT HERE

The effect of this modification (Figure 11 and 12) goes in the predicted direction: selection is less strong. In the Simulation F (Figure 11, with $\beta = 0.4$), results are similar to the “Standard Simulation”. In this formulation, in house R&D and cooperative R&D are complementary: starting cooperative projects increases the value of in-house R&D, and the presence of in-house R&D increases the incentive of cooperative R&D through its effect on size. Then the two effects, strengthening and weakening the selection process, substantially cancel out.

In the second case (Simulation G), the effect of decreased concentration is stronger. In this case (Figure 12), $\beta = 0.2$ and $k=0.5$, which means that in-house R&D is equivalent to a collaboration with a firm having the same technological position and efficiency 0.1. However, the main point here is that the results of the model respond “smoothly” to a limited ability of firm to progress autonomously in cost reduction: the logic in the arguments put forth in the previous sub-section is still valid.

5. Conclusion

In this paper we presented a model of dynamic R&D network formation, in which the focus is explicitly on the joint dynamics of market structure, firms’ technological capabilities and network evolution. First, through an analytical experiment, we spelled out more clearly the effects of market competition and technological opportunities on the firms’ incentive to collaborate. Then, we showed results from numerical simulations. They show the general importance of R&D networks as powerful selection

mechanism, leading firms that are not able to join the network or that occupy weak positions to exit the market. Both the selection process and the evolution of the network structure are driven by a self-reinforcing, path-dependent process, in which events in the early stages of industry affect firms' centrality in the initial network with long term consequences in terms of survival. At the same time, in the long run the network levels out the differences among the surviving firms, through a process of "densification" of the network that leads to the emergence of a complete or almost complete network in the phase of industry maturity. Also, we showed how the rate of technical progress, in the form of "high technological opportunity" or availability of partners, can affect the industry structure in the long run; we pointed out the detrimental effect on innovation generated by a slow process of technological converge among firms; we showed that the model is robust to the proposed modifications of the network formation algorithm and to a simple introduction of in-house R&D.

From the empirical point of view, the model is consistent with some evidence on the evolution of R&D networks. In particular, the model provides an economic, micro-founded rationale for the "preferential attachment mechanism" (i.e. highly central firms become more central in the subsequent periods), which is a recurrent feature in networks of technological agreements [Powell *et al.*].

Systematic analyses of the role and effect of R&D network on industry evolution are still missing. However, the model seems consistent with the appreciative argument on the emergence of "knowledge-based networked oligopolies" [Delapierre and Mytelka, (1998)]. In sectors like pharmaceuticals and ICT, a denser and denser network is emerging, involving the big players at the global level¹³. Furthermore, this web of

¹³ However, there is some evidence that for pharmaceuticals the rapid formation of alliances has stopped, starting in 1995.

alliances constitutes a significant barrier to entry (in the model, a barrier to survival), when rapid technical progress and strong competition make impossible a stand-alone strategy.

In the future, we plan to extend the analysis in two directions. First, as already mentioned in the paper, we want to consider the role of new entrants in the coevolution of industry and network. Second, from a policy perspective, we intend to investigate the effects that policies to promote R&D cooperation have on industries evolution, technological progress and social welfare.

Appendix

Proof of Proposition 1

Simple derivations show that:

$$\frac{\partial F}{\partial d(i, j)} = \lambda(a_2 - a_3 d(i, j))c \left[\frac{n\gamma_j(1 - \gamma_i)(e^{-\lambda(\gamma_j f(d(i, j)))} - 1) - \gamma_i(1 - \gamma_j)(1 - e^{-\lambda f(d(i, j))\gamma_i})}{n + 1} \right]$$

As long as firms are close enough in efficiency levels, the second factor is positive¹⁴. The sign of the derivative is then determined by $a_2 - a_3 d(i, j)$, which is positive if firms' distance is lower than the optimal one, and negative otherwise.

Proof of Proposition 2

Deriving one obtains:

$$\begin{aligned} \frac{\partial F}{\partial \gamma_j} = & c \left[\frac{n\lambda f(d(i, j))(1 - \gamma_i)e^{-\lambda\gamma_j f(d(i, j))} + 1 - e^{-\lambda\gamma_i f(d(i, j))}}{n + 1} \right] [q_i(+ij) + q_i(-ij)] \\ & + c \left[\frac{(1 - \gamma_j)(e^{-\lambda f(d(i, j))\gamma_i} - 1) - n(1 - \gamma_i)(e^{-\lambda\gamma_j f(d(i, j))} - 1)}{n + 1} \right] \left[\frac{dq_i(+ij)}{d\gamma_j} + \frac{dq_i(-ij)}{d\gamma_j} \right] \end{aligned}$$

¹⁴ Notice however that the condition of positivity here is stricter than the necessary condition of positive gains from collaboration.

The quantities in the first two square brackets are positive, so it is the first addend. The sign of the second addend depends on

$$\frac{dq_i(+ij)}{d\gamma_j} + \frac{dq_i(-ij)}{d\gamma_j} = \frac{c[\lambda f(d(i, j))n(1-\gamma_i)e^{-\lambda\gamma_j f(d(i, j))} - e^{-\lambda\gamma_i f(d(i, j))} - 1]}{n+1}$$

which is negative for λ sufficiently small.

From the study of the second derivative, it can be shown that it is negative for λ sufficiently small. Then the point (if any) where the derivative becomes 0 must be a maximum point. If gains from the collaboration are positive, there are consequently three possible cases: the increase in γ_j 1) has always a positive effect; 2) has always a negative effect; 3) has a positive effect initially, and then has a negative effect.

Proof of Proposition 3

Deriving one obtains:

$$\begin{aligned} \frac{\partial F}{\partial \gamma_i} = & c \left[\frac{-\lambda f(d(i, j))(1-\gamma_j)e^{-\lambda\gamma_i f(d(i, j))} + n(e^{-\lambda\gamma_j f(d(i, j))} - 1)}{n+1} \right] [q_i(+ij) + q_i(-ij)] \\ & + c \left[\frac{(1-\gamma_j)(e^{-\lambda f(d(i, j))\gamma_i} - 1) - n(1-\gamma_i)(e^{-\lambda\gamma_j f(d(i, j))} - 1)}{n+1} \right] \left[\frac{dq_i(+ij)}{d\gamma_i} + \frac{dq_i(-ij)}{d\gamma_i} \right] \end{aligned}$$

The first addend is negative, while, if the necessary condition for positive gain holds, the sign of the

second addend depends on $\left[\frac{dq_i(+ij)}{d\gamma_i} + \frac{dq_i(-ij)}{d\gamma_i} \right]$.

It can be shown that:

$$\left[\frac{dq_i(+ij)}{d\gamma_i} + \frac{dq_i(-ij)}{d\gamma_i} \right] = c \left[\frac{n(1 + e^{-\lambda\gamma_j f(d_{ij})}) - (1 - \gamma_j)\lambda f(d(i, j))e^{-\lambda\gamma_j f(d(i, j))}}{n + 1} \right]$$

The first quantity in square brackets is larger than 1, while the second is smaller than 1 for $\lambda f(d(i, j))$ small. Their difference is then positive.

The overall effect is ambiguous. Studying the second derivative, one gets $\frac{\partial^2 F}{\partial \gamma_i^2} < 0$ for λ sufficiently small. Then the point (if any) where the derivative becomes 0 must be a maximum point. There are consequently three possible cases: the increase in γ_i 1) has always a positive effect; 2) has always a negative effect; 3) has a positive effect initially, and then a negative effect.

Proof of Proposition 4

The proposition comes directly from:

$$\frac{\partial F}{\partial \sum_{k \neq i, j} c_k} = \frac{2}{n + 1} \left[\frac{(1 - \gamma_j)(e^{-\lambda f(d(i, j))\gamma_j} - 1) - n(1 - \gamma_i)(e^{-\lambda\gamma_j f(d(i, j))} - 1)}{n + 1} \right]$$

Proof of Theorem 1

We consider the situation where a stable oligopolistic structure has emerged, in the sense that the number of firms will remain constant in the future (the market structure at time t will be maintained in all the

subsequent periods if $\frac{A - n_t c_{it}}{n_t + 1} > 0 \quad \forall i \in N_t$).

We have to prove that $\lim_{t \rightarrow \infty} \Pr(g_{ijt} = 1) = 0 \forall ij \in N_t^2$. If $\lim_{t \rightarrow \infty} \Pr(g_{ijt} = 1) \neq 0$ we would have $\lim_{t \rightarrow \infty} \gamma_i = \lim_{t \rightarrow \infty} \gamma_j = 1$. By continuity of $F(\cdot)$ (which is the gain function defined in section 4.1), this implies $\lim_{t \rightarrow \infty} F_i(\alpha_j, \gamma_j) = \lim_{t \rightarrow \infty} F_j(\alpha_i, \gamma_i) = 0$. But then, since $E > 0$, the link will asymptotically become unprofitable. Given that each link is updated with a positive probability, it will be severed with probability 1 as $t \rightarrow \infty$, and then we have the initial claim.

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Figure 1: Gains from collaboration-1

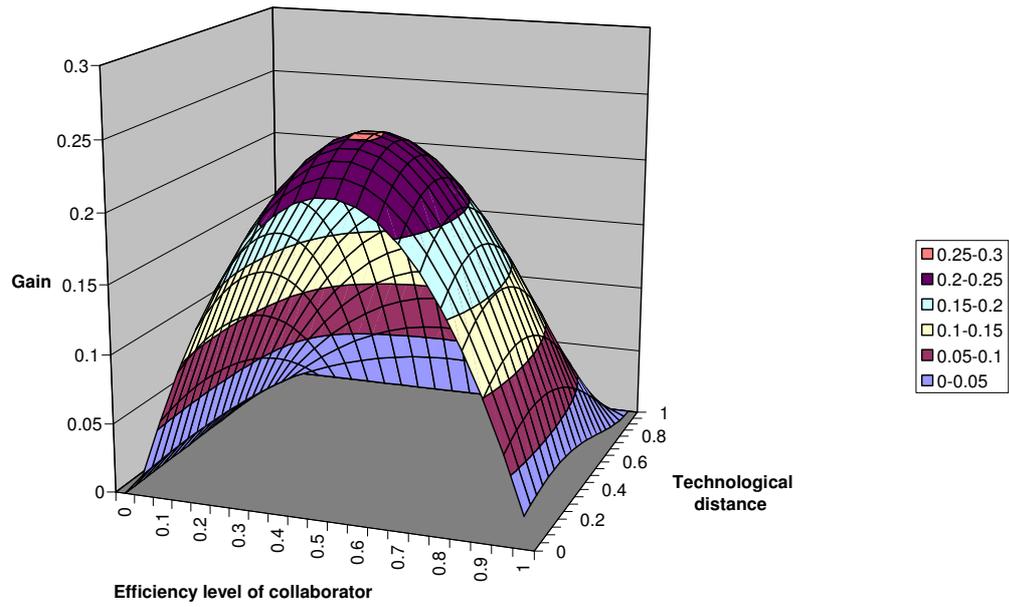


Figure 2: Gains from collaboration-2

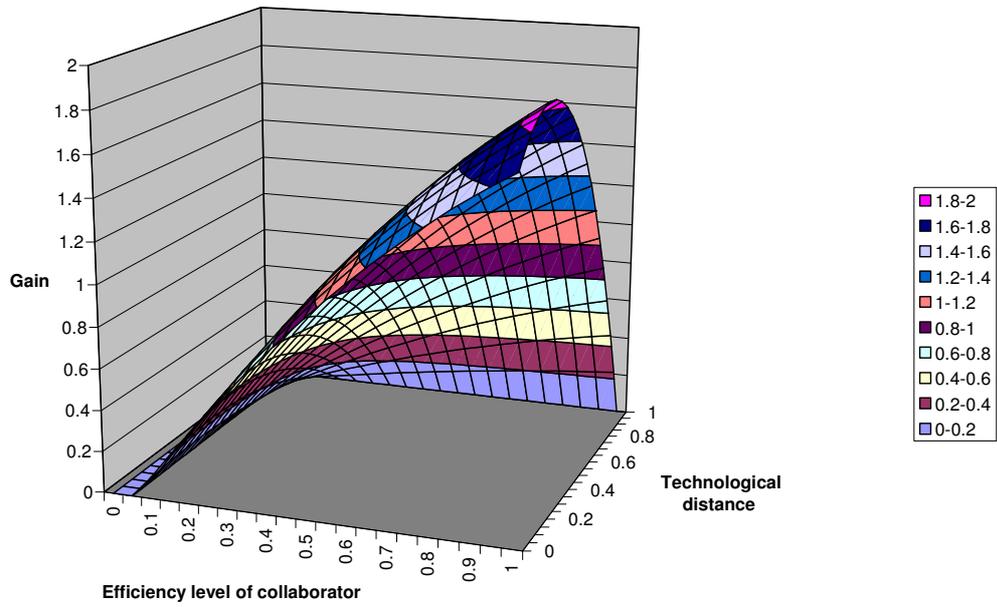


Figure 3

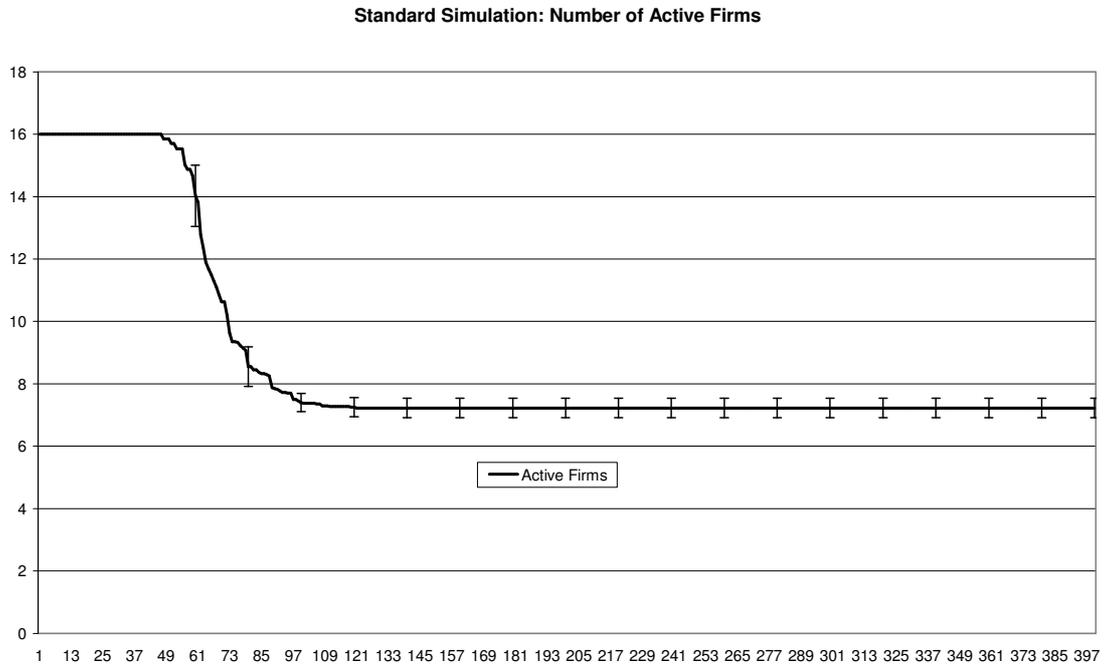


Figure 4

Standard Simulation: Total Quantity, Herfindhal Index and Network Density

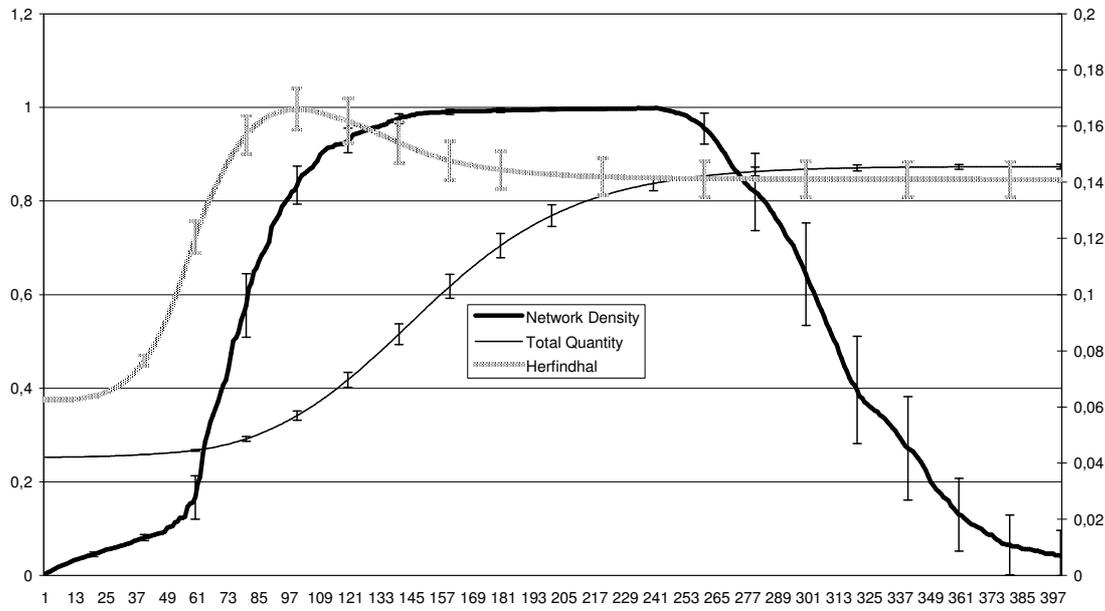


Figure 5

Standard Simulation: Degree Centralization

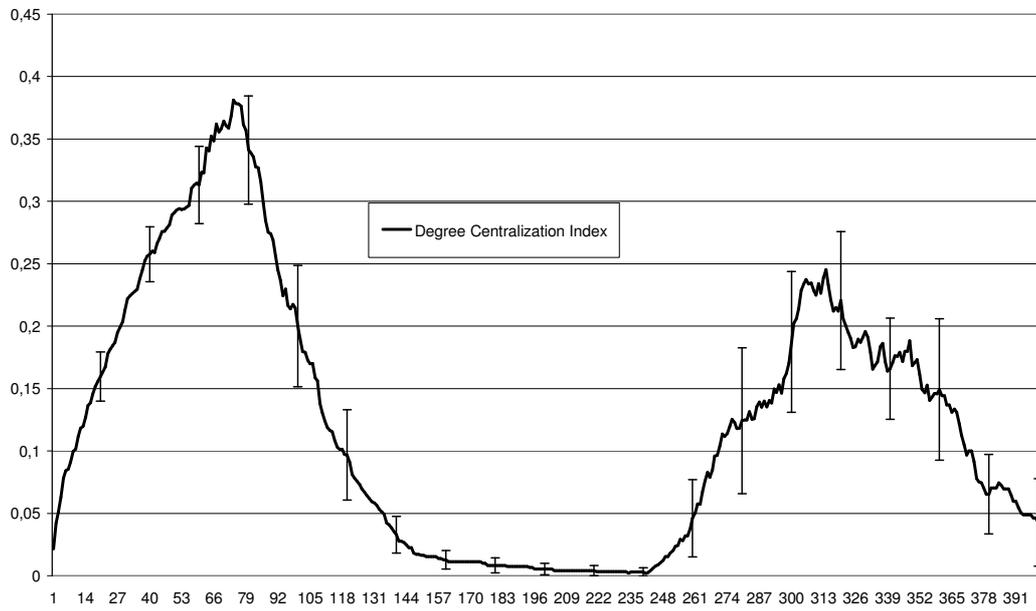


Table 1

	<i>Surviving</i>		
<i>Link40</i>	0	1	<u>Total</u>
0	303	3	306
1	48	286	334
<u>Total</u>	351	289	

Table 2

<i>newlink40</i>	<i>Coeff.</i>	<i>Std. Err</i>	<i>t</i>	<i>P> t </i>
<i>newcalled</i>	0.2363637	0.0195672	12.08	0.000
<i>link10</i>	0.7089678	0.0574208	12.35	0.000
<i>constant</i>	0.3405201	0.0848473	-4.01	0.000
<i>Number of obs</i>	640			
<i>F(2,637)</i>	149.58			
<i>R-squared</i>	0.3196			

Figure 6

Simulation A: Total Quantity, Herfindhal Index and Network Density

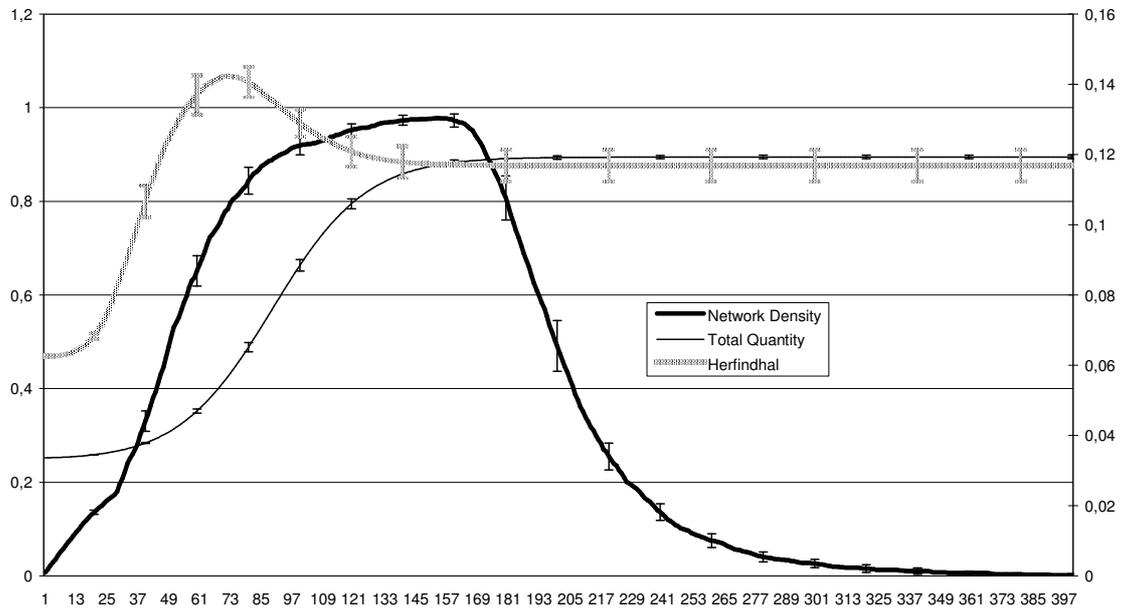


Figure 7

Simulation B: Total Quantity, Herfindhal Index and Network Density

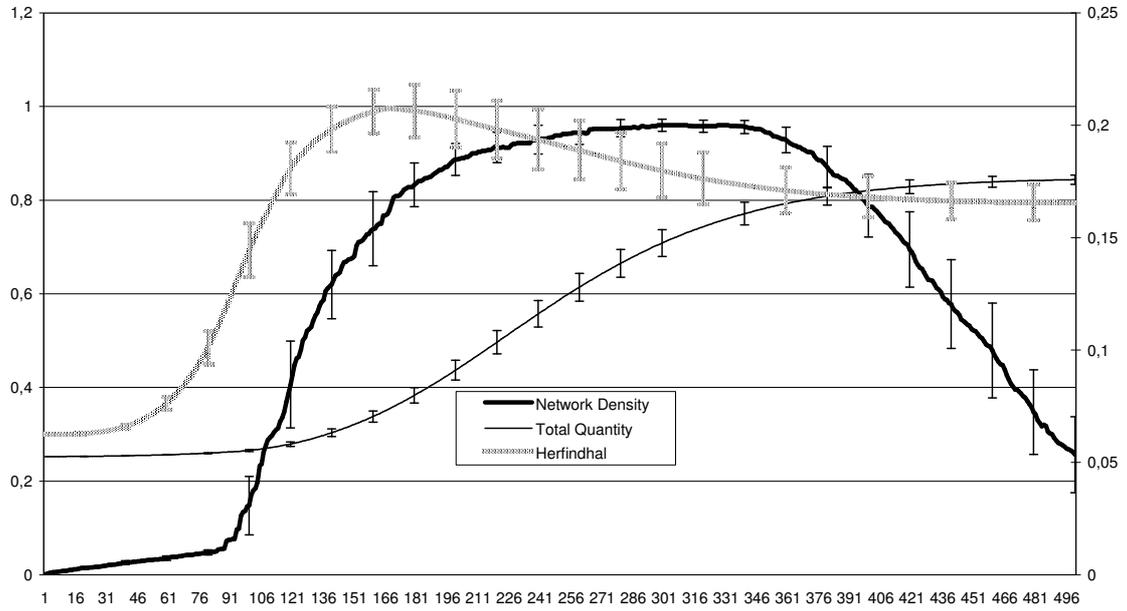


Figure 8

Simulation C: Total Quantity, Herfindhal Index and Network Density

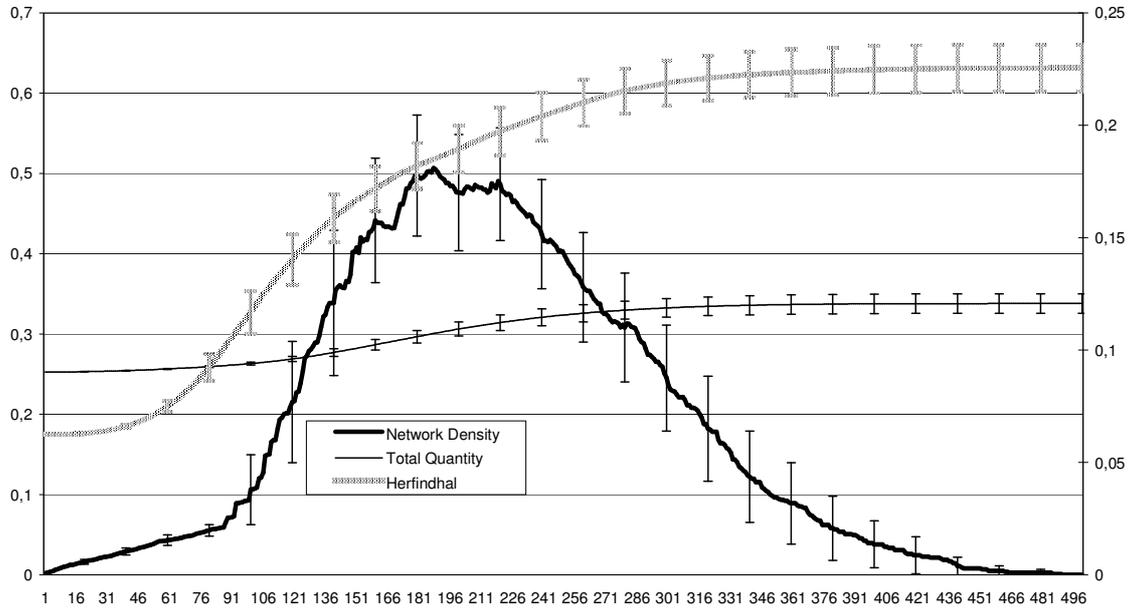


Figure 9

Simulation D: Total Quantity, Herfindhal Index and Network Density

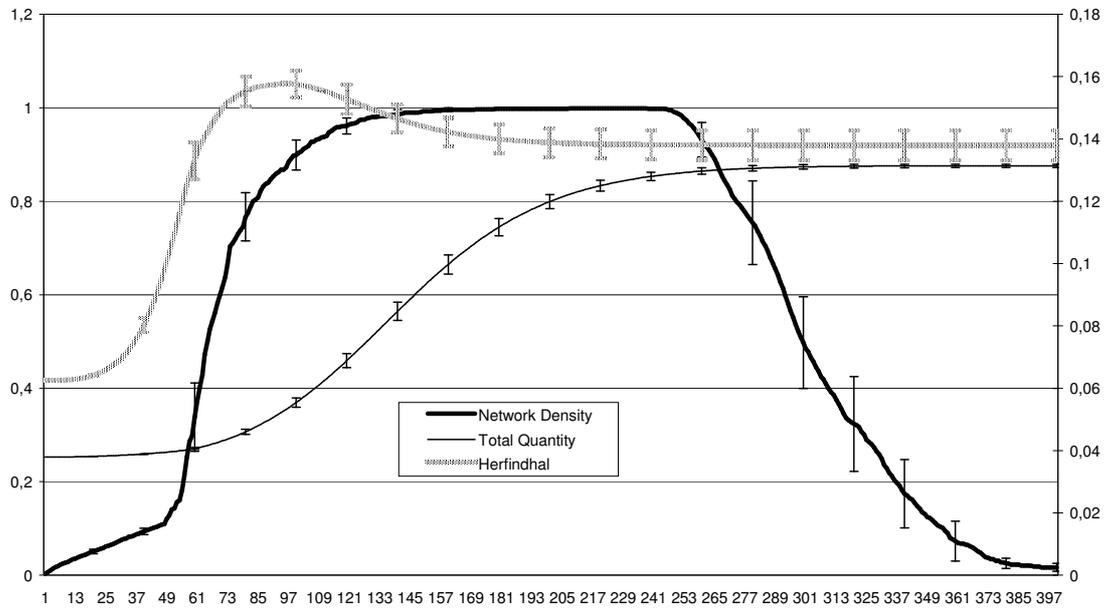


Figure 10

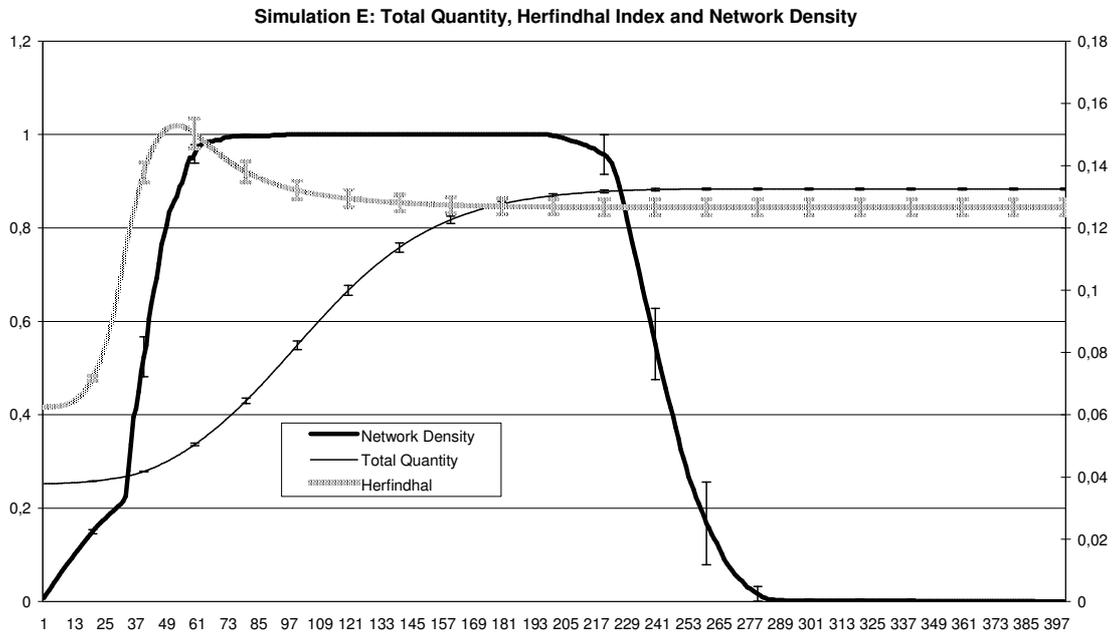


Figure 11

Simulation F: Total Quantity, Herfindhal Index and Network Density

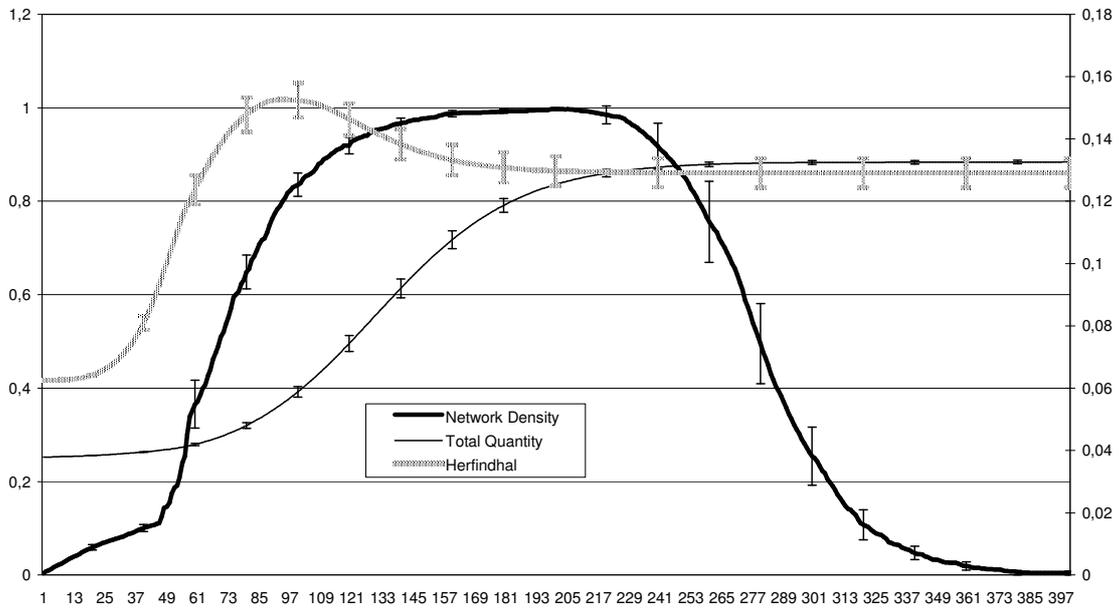


Figure 12

Simulation G: Total Quantity, Herfindhal Index and Network Density

