



Paper to be presented at
DRUID15, Rome, June 15-17, 2015
(Coorganized with LUISS)

Transitions, percolation and critical fragmentation.

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Abstract

A technological transition takes place when a new technology diffuses in a population where the old paradigm is dominant and benefits from increasing returns to adoption. Traditional explanations that require a critical mass of adopters of the new technology to trigger a regime shift. This paper suggests an explanation based on a critical fragmentation. When the population is fragmented between several technologies (among them the old regime), the increasing returns to adoption for the old regime is weakened. A new technology at that point can gather the adopters of the previously failed technologies, as well as those from the old regime, and become the new dominant paradigm. This hypothesis is analyzed by means of a simulation model of repeated diffusion processes for a population embedded in a social network. Diffusion is modeled with the percolation framework, extended with increasing returns to adoption. All technologies have the same intrinsic value, although agents update their perceived value of a technology depending on the number of fellow adopters in their local environment, compared to those that still remain in the old regime.

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1 Introduction

Technological and societal transitions are generally understood as a shift in the dominant paradigm (Geels, 2010). A new paradigm has to oust the existing, established old paradigm in order to become dominant, overcoming path dependence and the almost irreversible character of technological development. Understanding the triggers of a regime shift is key to stimulate a sustainability transition (Zeppini et al, 2014).

From a diffusion perspective, the new paradigm has to diffuse in a population where the old paradigm is dominant and benefits from increasing returns to adoption. A critical mass of adopters is usually needed for a population to switch from an old paradigm to a new one (Arthur, 1989; Bruckner et al, 1996). This critical mass can come from several sources, like a market niche or coordination between agents. Thus, a traditional explanation for a paradigm shift is a high concentration of adopters of the new paradigm in the overall population.

Nonetheless, a paradigm shift generally requires several tries to be successful. That is say, several candidate paradigms fail to become dominant, before one successfully replaces the old paradigm. A usual justification is that the process of trial-and-error echoes an exploring process where new candidate paradigms learn from the failure of the previous ones. The candidate paradigm that replaces the old regime is assumed to be better than the ones who failed and good enough to become dominant.

In this paper we suggest that new technologies that succeed in replacing a dominant one might not be better than previously failed attempts, but might just arrive at the right time. Every new technology diffuses to a small amount of people, gradually shattering the ground of advocates of the old regime. Due to increasing returns to adoption, agents experience a social reinforcement in adopting the same technology of their social contacts. After several technologies have failed to replace the old regime, the population is fragmented between several candidate paradigms and the old regime: the social reinforcement for the old paradigm is weakened. A new technology that comes in that moment can find the right conditions to diffuse and replace the old regime, without being intrinsically better than the previous ones. The triggering event of transitions is then a “critical fragmentation” of adopters, rather than a “critical mass”, which characterises more traditional explanations of technological regime shift.

We analyze our hypothesis with a model of repeated diffusion processes for a population embedded in a social network. We model diffusion in a percolation framework (Solomon et al, 2000) that represents a word-of-mouth communication in a social network (Campbell, 2013). All technologies have the same intrinsic value, although the population perceives them as different due to the social influence of increasing returns to adoption. Agents update their perceived value of

a paradigm with its diffusion among their friends compared to the number of friends that still remain in the old regime. Thus, diffusion is driven by local social influence.

We find several conditions under which our model reproduces realistic patterns of regime shift. The first condition concerns the intrinsic value of technologies. If this value is too low, a new technology can never become dominant. On the other hand, technologies with a very high intrinsic value will immediately replace the old regime, without a need for previous failed trials, as their intrinsic value is high enough to compensate the increasing returns to adoption of the old regime. The most realistic scenario of several attempts preceding a successful transition requires that such intrinsic value be near to the percolation threshold of the network.

A second condition is that for a transition to occur new technologies need to have some advantage over older ones in order to attract adopters of failed technologies. Otherwise, the population remains fragmented over different technologies, none of which can become dominant. Finally, the diffusion process needs a moderate level of social reinforcement or increasing returns to adoption. If social reinforcement is too low, the number of advocates of a technology is irrelevant for its diffusion. On the other hand, too much social reinforcement can lead to a herd movement where all the adopters of a technology jump to the next arriving technology, without allowing for a fragmented population. This result depends on the intrinsic value of technologies, and only holds when the intrinsic value is on the threshold of the percolation process.

In conclusion, this paper posits a new explanation of technological transitions under increasing returns to adoption which is alternative with respect to traditional arguments based on trial-and-error or critical mass. We suggest that a new technology becomes dominant when arriving at the right moment, instead of being better than failed attempts. The right moment consists of a social base which is sufficiently fragmented among different competing options. Such fragmentation causes a shift of the percolation threshold of the network towards lower values. The trigger of regime shifts is thus a “critical fragmentation” of the population, rather than a critical mass of adopters of the new technology.

The paper is organised as follows. Section 2 reviews previous explanations of transitions in presence of increasing returns to adoption. Section 3 presents the percolation model with increasing returns to adoption. Simulations of the model are analyzed in Section 4. Finally, Section 5 offers some concluding remarks.

2 Traditional explanations for transitions

A transition is a fundamental system change that involves changing the dominant paradigm for a new paradigm. This paradigms usually benefit form increasing returns to adoption, that is

to say, that the benefit of adopting a technology for an individual increases with the number of fellow adopters. Increasing returns to adoption have many origins, including knowledge spillovers, economies of scale, network externalities, and learning by doing on the side of users (Frenken et al, 2004). Under increasing returns to adoption, a transition requires escaping lock-in of the present technological system, that already counts with a great mass of adopters (Alkemade et al, 2009).

Scientific literature on transitions has dealt with the determinants of a technological substitution. Under increasing returns to adoption, the “fitness” of a technology is not intrinsic to the technology, but also depends on its frequency in the population. A new technology with a higher intrinsic fitness cannot compete with an incumbent technology with a lower intrinsic fitness but a much bigger pool of adopters. Evolutionary models have emphasized this possibility that the system becomes locked-in in a suboptimal technology (David, 1985; Arthur, 1989). In such a case, triggering a transition can be a political question.

Starting with Bruckner et al (1996), evolutionary models have tried to identify the conditions for a transition under increasing returns. Bruckner et al (1996) found that no new technology could succeed if it started with a small number of adopters. This critical mass of adopters of the new technology is usually required to trigger a technological substitution, although how this critical mass is created can change.

When technologies are heterogeneous, they can compete in some performance characteristic. In such a case, a new technology may successfully be introduced in a market niche. This niche can protect it while it develops, until it has acquired a sufficient number of adopters to compete with the old technology (Frenken et al, 2004).

On the other hand, agents that are not myopic can foresee the benefit of introducing a new technology and collecting the benefit of increasing returns to adoption. They can coordinate their decision to adopt the new technology, if they recognize that this coordination will report important benefits (Lissoni, 2005). These coordinated adopters provide with a critical mass to substitute the old technology and trigger a transition.

This paper offers an alternative determinant for transitions. A new technology that enters the market and is not successful can still steal some of the adopters of the old technology. After several failed technological substitutions, the old paradigm has lost part of its adopters, and the population is in a state of “critical fragmentation”. No technology retains high benefits from the number of adopters, as the population is divided in their choice of technologies. Under these conditions, a new technology can find the ground prepared to diffuse both among adopters of the failed technologies, as among the remaining adopters of the old technology. This explanation differs from the others in that the critical fragmentation does not bring a critical mass of adopters of the new paradigm, but a critical mass of un-adopters of the dominant paradigm.

3 The model

3.1 Percolation

We analyze the diffusion process of new technologies on a population that presents a social network structure. technologies are identified by their value, represented by a number $v \in [0, 1]$. Agents are heterogeneous in their incredulity, or resistance to adopt the technology.¹ They are characterized by their minimum quality requirement (MQR) for adopting a new technology. The higher the MQR -the more incredulous an agent is- the higher the value he requires of an technology in order to adopt it. In this section, the MQR of agents is a random variable which is uniformly distributed, $q \sim U[0, 1]$. In the next section we will change this distribution to non-uniform.

This modelling framework corresponds to the so-called percolation model (see Solomon et al, 2000). In a percolation model of diffusion time is discrete. One agent adopts the new technology at any given time t if the following three conditions are met:

- the agent has not adopted before t ,
- the agent is informed, which only occurs if at least one neighbor has adopted at time $t - 1$,
- the value of the technology is higher than the MQR of the agent, that is $q < v$.

Without a social structure the percolation model behaves as a well-mixed population of consumers. In a well-mixed population, agents are not embedded in a social network and they have perfect information. As soon as the technology enters the “market”, the willing to adopt agents adopt it while the rest do not. As the MQR is uniformly distributed $q \sim U[0, 1]$, a proportion $100 \cdot v_0\%$ of the population will adopt an technology of value $v_0 \in [0, 1]$. This case can be represented in our model with a complete network, where every agent is connected to every other agent. In a complete network, a single early adopter will inform the whole population of agents about the existence of the technology.

3.2 Social network

In a percolation setting, agents become informed of the existence of the technology through their neighbors. Thus, the structure of the social network where the agents are embedded can be determinant of the outcome of the process. Previous studies have considered percolation processes in regular networks as a two dimensional lattice (Hohnisch et al, 2008; Cantono and Silverberg, 2009; Zheng et al, 2013) or a completely random network (Campbell, 2013).

¹In an epidemic model, this incredulity would be equivalent to the resistance to contagion.

In this paper we propose the use of the small world algorithm (Watts and Strogatz, 1998) for the modelling of the social structure as in Cowan and Jonard (2004). This provides with a family of networks, an interpolation between regular lattices and completely random networks. The algorithm starts with a regular ring lattice and rewires every link with probability μ . This parameter allows to fine tune the randomness of the network. Figure 1 shows the result of simulating a percolation process in small worlds with different rewiring probabilities $\mu \in 1, 0.1, 0.01, 0.001, 0$.

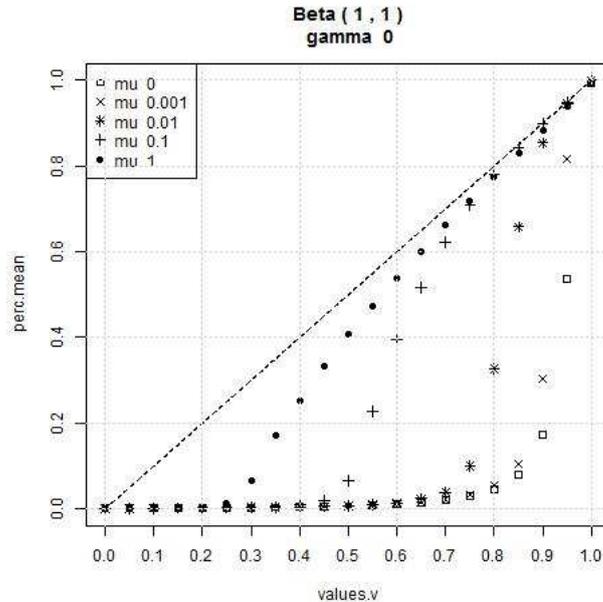


Figure 1: Percolation in different network structures

The clustering coefficient of a network is the relative number of triads present in the network. Varying the rewiring probability μ of the small world algorithm produces networks with varying average path length and clustering coefficient (Figure 2). The case with $\mu = 0$ is the one-dimensional regular lattice, and the case with $\mu = 1$ is the random network, also known as Poisson network or Erdos-Renyi model (Erdos and Renyi, 1959). For intermediate probabilities μ , the resulting networks present intermediate clustering coefficients. The “typical” small world is the one with rewiring probability $\mu = 0.01$, presenting an average path-length almost as low as the Poisson network, while still having a clustering coefficient which is comparable with the one-dimensional regular lattice. This network structure will be used for the results.

3.3 Increasing returns to adoption

So far, the percolation process described is a simple propagation. Only the first time the agents are informed about the idea determines whether they adopt it or not, additional contacts are redundant. Here we extend the basic percolation model with social reinforcement or local increas-

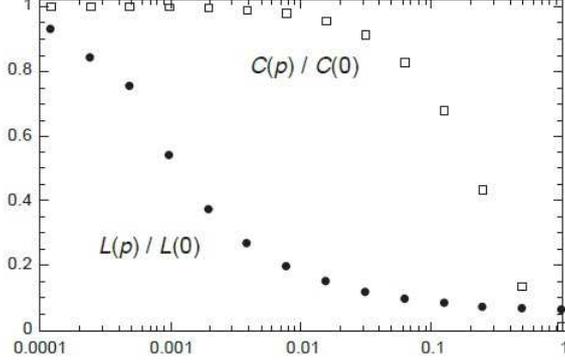


Figure 2: Clustering coefficient C and average path-length L as a function of the rewiring probability in small world networks (Source: Watts and Strogatz (1998)).

ing returns to adoption. With social reinforcement, additional contacts will increase an agent's willingness to adopt.

The difference between the basic percolation model and the increasing returns to adoption extension lies in how the MQR of agents is calculated. Let q_t be the MQR of an agent at time t . In the basic percolation model this threshold remains constant over time, with $q_t = q_0 \forall t$. Thus, the number of adopting neighbors does not play any role in adoption decisions. We include a new factor in the expression of the value of a technology, according to which decisions are influenced by the number of adopting neighbors.

$$q_t^i = q_0^i \cdot \left(\frac{1}{a_t^i + \phi \sum_{j=1}^{i-1} a_t^j} \right)^\gamma \quad (1)$$

The updated MQR is defined to satisfy the following hypothesis of the model. Let $q \in [0, 1]$ be the MQR of an agent, $a \in \mathbb{N}$ the number of adopting neighbors and $\gamma \in [0, 1]$ a parameter expressing the increasing returns to adoption intensity. The functional form $f(q, a, \gamma)$ is chosen such that: (1) it is decreasing in the number of adopting neighbors, $\frac{\partial f}{\partial a} < 0$, so that the more neighbors adopt, the easier it is for an agent to adopt; (2) it is decreasing in the intensity of the increasing returns to adoption, $\frac{\partial f}{\partial \gamma} < 0$, so that with the same number of adopting neighbors, the updated value of MQR will be lower for higher γ ; (3) with only one neighbor adopting it is equal to the initial MQR q_0 ; (4) in the absence of increasing returns to adoption ($\gamma = 0$) it is equal to the basic percolation model. Finally, the different technologies are all competing against the same old regime, so they are alternative technologies. Parameter $\phi \in [0, 1]$ measures the effect of neighbors who have abandoned the original regime: $\phi = 0$ means that the returns to adoption for technology j are only increased by adopters of the same technology j ; while $\phi = 1$ means that the returns to adoption for technology j are increased by all the previous technologies different from the old regime, at the same rate. The functional form in Equation (1) fulfills all four conditions.

$$q_t^i = q_0^i \cdot \left(\frac{1}{a_t^i}\right)^\gamma \quad (2)$$

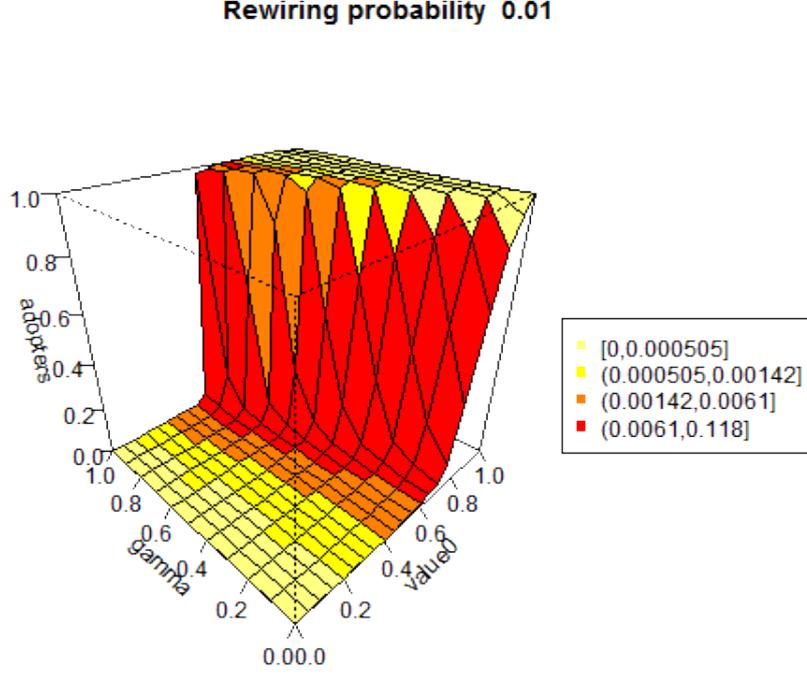


Figure 3: Diffusion of a technology, depending on the intrinsic value v_0 and the increasing returns to adoption intensity γ

For a single technology competing against the old regime, Equation (1) is equivalent to Equation (2). Figure 3 shows the mean portion of agents that will adopt the new technology depending on the intrinsic value of the technology v_0 and the increasing returns to adoption intensity γ , over 20 Monte Carlo runs. The colors show the standard deviation over the different runs. The zones with higher standard deviation show the thresholds between the diffusion and the non-diffusion regimes. We will use parameter values in these thresholds, where the result of the process is most unpredictable a priori.

4 Results

In this section we study the percolation model extended with increasing returns to adoption by mean of batch simulation experiments. Technologies diffuse in a small world network of $N = 10,000$ nodes representing potential adopters, with $k = 4$ neighbors on average and rewiring

probability $\mu = 0.01$. We simulate the model in different settings represented by the intrinsic value of the technologies v_0 , and the social reinforcement intensity γ . The MQRs of agents are random draws from a uniform distribution, $q \sim U[0, 1]$.

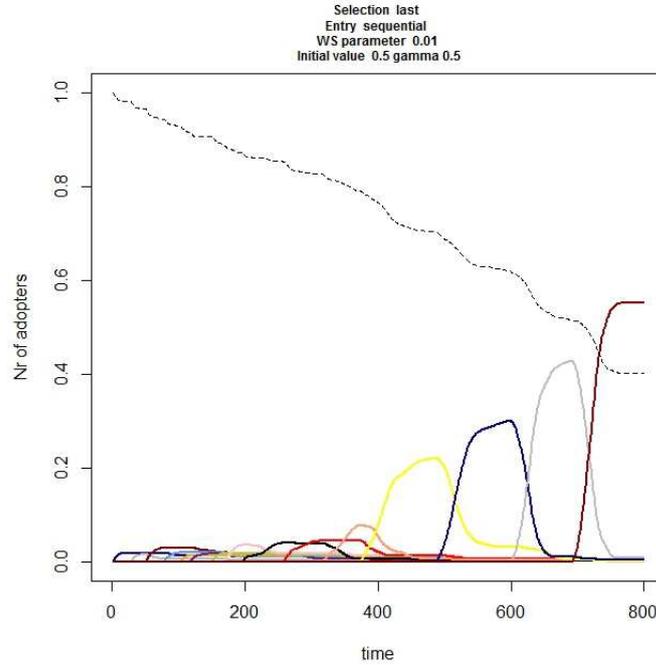


Figure 4: Critical fragmentation triggers diffusion. Last technology selection, $v_0 = 0.5, \gamma = 0.5$

Figure 4 shows the result of simulating the process with an intrinsic value of $v_0 = 0.5$ and increasing returns to adoption intensity $\gamma = 0.5$. This combination of parameters is in the threshold of the diffusion and the non-diffusion regimes shown in Figure 3. Every line represents the percentage of adopters of a different technology that tries to replace the old regime. For the first half of the simulation period, many technologies enter the population and fail to diffuse. They all behave similarly, getting to a similar portion of the population. At around period $t = 400$, a new technology enters the population that fares significantly better than the rest (the yellow line). This technology marks the beginning of a different pattern of diffusion. From that point on, every new technology diffuses to a higher portion of the population than the last, collecting adopters both from earlier technologies as from the old regime. Finally, the last technology becomes dominant in the population and only competes against the old regime.

One of the conditions of this model is that agents can only be adopters of one technology at a time, either a new technology, either the old regime. When they face a tie, that is to say, when an agent can adopt two or more technologies, they can have two rules of decision. In the first case, they will choose their favorite technology, based on the intrinsic value of the technology, the number of neighbors adopters of that technology and the rest, and their MQR for that technology.

Thus, technologies that arrive earlier in the simulation have an advantage over later arrivals, as they have already a pool of adopters that will increase their perceived value. In the second case, agents choose the last one to enter the population, removing this advantage of earlier technologies. In the scenario depicted by Figure 4, an agent that can adopt two technologies will choose the newest one.

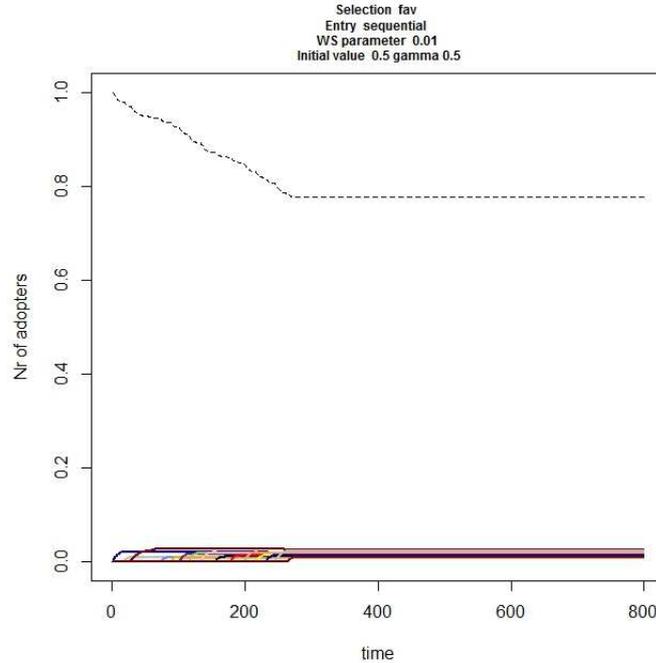


Figure 5: When older technologies have an advantage, diffusion is hampered. Favorite technology selection, $v_0 = 0.5, \gamma = 0.5$

The other scenario, when older technologies have an advantage over newer arrivals, is depicted in Figure 5. In this case diffusion is interrupted. Agents can only adopt a technology that they just became informed about, that is to say, that a neighbor just adopted in the previous step. Thus, additional adopters of different technologies do not change the benefits of adopting an earlier technology that agents cannot adopt anymore. Every new arriving technology diffuses in only a small portion of the population, but the effect of fragmenting the pool of adopters of the old regime cannot be collected.

The parameters for the intrinsic value v_0 and the increasing returns to adoption intensity γ interact in an intricate way. If the value is too low, the technologies will not diffuse, and if γ is too low, the number of fellow adopters will not affect the process. On the other extreme, if the value is very high, every new technology will replace the old regime (and the previous technologies). A case with high value of γ is depicted in Figure 6. This scenario has the same intrinsic value as the other two, $v_0 = 0.5$, but a higher increasing returns to adoption intensity, $\gamma = 0.8$. This combination of

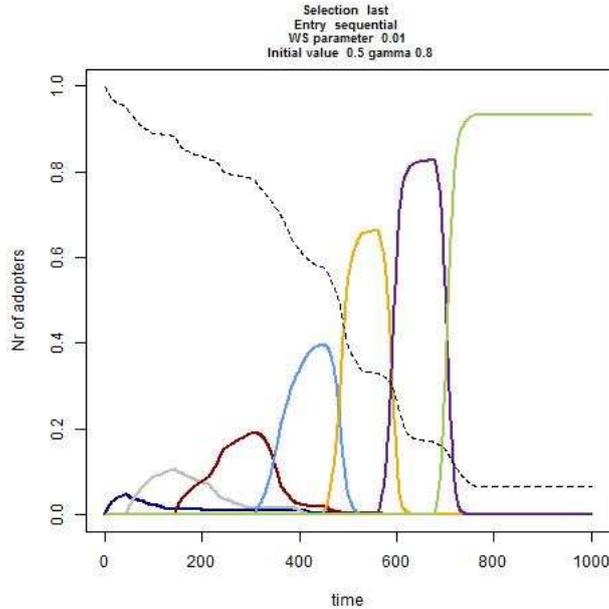


Figure 6: When increasing returns to adoption are high, new technologies replace the previous candidates. Last technology selection, $v_0 = 0.6, \gamma = 0.7$

parameters is still in the threshold of the diffusion and the non-diffusion regimes shown in Figure 3. With such a high effect of fellow adopters, the population cannot stay fragmented. Every new technology will replace the previous one and diffuse to a bigger portion of the population. This behavior is similar to that on the second part of the first case (Figure 4). In the first scenario increasing returns to adoption built slowly for the first half of the simulation. After the critical fragmentation was reached, every new technology to arrive would collect the adopters of older technologies and the old regime. In this case, increasing returns to adoption are already high enough for every new technology to gather all the adopters of earlier technologies, as well as some additional from the old regime.

5 Conclusions

During a technological transition, a new technology takes the place of an already existing dominant paradigm. A transition can be analyzed from a diffusion angle, considering that a new technology has to diffuse in a population where the old paradigm is dominant and benefits from increasing returns to adoption. Traditional explanations of transitions under increasing returns to adoption usually require a critical mass of adopters of the new technology to trigger a regime shift. Such a critical mass can form in a market niche, or from coordination between agents.

This paper suggests an alternative explanation of transitions under increasing returns to adoption. Instead of a critical mass of adopters, that is to say, a critical level of coordination against

the old regime in the population, we consider the effect of a critical fragmentation, that is to say, a critical level of dis-coordination against the old regime. Every new technology that fails to replace the dominant paradigm, gradually shatters the pool of adopters of the old regime. After several tries, the population is fragmented between several candidate paradigms and the old regime: the social reinforcement for the old paradigm is weakened. The conditions in such a case are propitious for a new technology to gather the adopters of the previously failed technologies, as well as those from the old regime, and become the new dominant paradigm.

This hypothesis is analyzed by means of a simulation model repeated diffusion processes for a population embedded in a social network. Diffusion is modeled with the percolation framework (Solomon et al, 2000), extended with increasing returns to adoption. All technologies have the same intrinsic value, although agents update their perceived value of a technology depending on the number of fellow adopters in their local environment, compared to those that still remain in the old regime.

The simulations show that critical fragmentation can shatter the increasing returns to adopting the old regime and build those favorable to the new technologies, under certain conditions. First of all, the intrinsic value of the technologies and the increasing returns to adoption intensity must be in the threshold between the diffusion and the non-diffusion regimes. That is to say, in the non-diffusion regime, none of the technologies that enter the population will diffuse, while in the diffusion regime, every new technology will replace the previous one (including the old regime) and become dominant. In the threshold, technologies will build on each other to replace the old regime.

Additionally, agents can only be adopters of one technology at a time, either a new technology, either the old regime. When facing a tie between two technologies, agents have to choose one of them. If they choose their favorite technology (the one they perceive as better), technologies that arrive earlier in the simulation have an advantage over later arrivals, as they have already a pool of adopters that will increase their perceived value. In this case, diffusion is interrupted. Agents can only adopt a technology that they just became informed about, that is to say, that a neighbor just adopted in the previous step. Thus, additional adopters of different technologies do not change the benefits of adopting an earlier technology that agents cannot adopt anymore. Every new arriving technology diffuses in only a small portion of the population, but the effect of fragmenting the pool of adopters of the old regime cannot be collected. For technologies to build on the increasing returns to adoption of previous ones, newer technologies need to have some advantage over earlier ones. Their intrinsic value, nonetheless, remains the same in all cases.

Finally, the intensity of increasing returns to adoption has to be moderate. If it is too low, the number of fellow adopters does not affect the process. On the other hand, if it is too high, the

population cannot stay fragmented. Every new technology replaces the previous one and diffuses to a bigger portion of the population. With a moderate intensity, increasing returns to adopting a new technology, and un-adopting the old regime, build slowly with technologies that fail to diffuse until they are high enough for a new technology to absorb the adopters of the previous ones and diffuse over those of the old regime.

In conclusion, this paper postulates a new explanation of technological transitions under increasing returns to adoption. We suggest that a new technology becomes dominant by arriving at the right moment. The right moment, in this case, is after several failed technologies have fragmented the social base of the old paradigm. Such fragmented population of adopters of different competing technologies causes a shift of the percolation threshold of the network towards lower values. The trigger of a regime shift is thus a “critical fragmentation” of the population, rather than a critical mass of adopters of the new technology.

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